

Thursday  
3 April

205

## Weak Interactions

Brief history of important events related to weak interactions

1896 Henri Becquerel discovered spontaneous radioactive decay, among the reactions was  $\beta$ -decay



1911 Lise Meitner & Otto Hahn showed  $\beta$ -decay violated conservation of energy

1930 W. Pauli hypothesized the existence of a neutral particle, emitted with the  $e^-$  in  $\beta$ -decay. This was motivated by the continuous energy spectrum of the  $e^-$ , and of course to restore energy conservation. He called it the 'neutron'

1932 Chadwick discovered the 'neutron' which was too massive to be Pauli's neutron

1934 Fermi developed the theory of  $\beta$ -decay, and coined Pauli's neutron, the neutrino (little neutron), to clean up the nomenclature.

- Nature rejected Fermi's paper detailing the theory of  $\beta$ -decay, calling it "too remote from reality."  
- so Fermi then became an experimentalist

1936  $\mu^-$  discovered by Anderson

1953  $\bar{\nu}e$  discovered in  $\bar{\nu}ep \rightarrow ne^+$

1956 - 57 Parity violation observed / developed  
Wu et al. 1957, Lee & Yang 1956

206

1958 V-A theory: Feynman & Gell-Mann, Guralnik, Hagen, Kibble

Sudarshan & Marshak

symmetries of Sakurai shows that gauge fields must be fermions

1961 Glashow develops  $SU(2) \times U(1)$  gauge theory of massive vector bosons

1962 Lederman, Schwartz & Steinberger discover  $\nu_e$

1963 Cabibbo proposed weak eigenstates not the same as mass (lepton or quark) eigenstates, explaining the

apparent differences in weak interactions in terms of a unified weak strength

1964 Higgs & many others propose Higgs mechanism of

1964 Gell-Mann & Zweig proposed quarks (aces)

Bjorken & Glashow predict charm quark

1967 Ray Davis measured significant missing solar neutrino flux (no one believed him for decades)

- Weinberg & Salam combine Higgs mechanism w/ Glashow's

$SU(2) \times U(1)$  gauge theory to explain the mass generation

of  $W, Z$  bosons

1971 't Hooft showed this Higgs gauge theory is renormalizable

1977 gauge anomalies shown to spoil this renormalizability

- UNLESS no. of lepton generations = no. quark generations

1973 KM show Cabibbo matrix contains CP violation if there are 3 flavors

1975  $\tau$  discovered ( $\nu_\tau$  strongly suspected)

1983 Carlo Rubbia & team discover  $W, Z$  bosons @ SPS @ CERN

2000  $\nu_e$  discovered

2012 Higgs discovered

↳ standard model + neutrino mass +  $F_2 \approx 32\%$

↓ 20%  $\mu \nu \gamma \gamma$  + 33%  $\gamma \gamma \gamma$  +  $F_{2\gamma} \approx 10\%$

Modern (V-A) version of Fermi's theory of Weak Interactions

"Charged Current Interactions" add (charge) to Fermi's theory

$$H = \frac{G_F}{\sqrt{2}} J_\mu^+ J^\mu$$

$$J_\mu^+ = J_{\mu, l}^+ + J_{\mu, h}^+$$

lepton  
current

hadronic  
current

$$\begin{aligned} J_{\mu, l}^+ &= \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e^- \\ &+ \bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu^- \\ &+ \bar{\nu}_\tau \gamma_\mu (1 - \gamma_5) \tau^- \end{aligned}$$

$$V-A \equiv \gamma_\mu (1 - \gamma_5)$$

The charged current interactions are maximally parity violating. (as opposed to  $\alpha \gamma_\mu - \beta \gamma_\mu \gamma_5$  with  $|\beta| \neq |\alpha|$ )

From Fermi's theory, the hadronic charged current

$$J_{\mu, h}^+ = \bar{p} \gamma_\mu (1 - \gamma_5) n \cos \theta_c \quad \theta_c = \text{Cabibbo angle}$$

We know there are many other hadronic weak decays

$$Z^- \rightarrow n$$

$$K^+ \rightarrow \pi^+ \nu$$

⋮

and we know about quarks. Before c, b, t quarks,

$$J_{\mu, h}^+ = \bar{u} \gamma_\mu (1 - \gamma_5) d' = 2 \bar{u}_L \gamma_\mu d'_L$$

$$q_L = \frac{1 - \gamma_5}{2} q$$

important role to play in the Cabibbo (A-V) mechanism

The important (crucial) observation Cabibbo made was that the Weak eigenstates need not be the same as the strong (mass) eigenstates.

$$d' = d \cos \theta_c + s \sin \theta_c$$

The strong flavor eigenstates are determined by the quark mass matrix:

we are free to perform global SU(3) rotations

$$\bar{\Psi}_f M_f \Psi_f = (\bar{u}, \bar{d}, \bar{s}) \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

But there is no reason the Weak eigenstates need to be in this same basis.

So the Weak flavor and strong (and lepton) flavor basis are rotated with respect to each other.

We now know  $\sin \theta_c \approx 0.23$

This observation allowed Cabibbo to show the leptonic and hadronic weak interactions are all given by a universal weak charge.

Ignoring all QCD  $\neq$  mass effects

$$|M(\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e)|^2 = |M(n \rightarrow p e^- \bar{\nu}_e)|^2 + |M(\Xi^- \rightarrow n e^- \bar{\nu}_e)|^2$$

$$\begin{array}{ccc} \mu \rightarrow e & d \rightarrow u & s \rightarrow u \\ 1 & = & \cos^2 \theta_c + \sin^2 \theta_c \end{array}$$

Before the discovery of the  $\tau$

$$J_{\mu, e}^+ = \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e + \bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu$$

To make the hadronic current look more like the lepton current, Bjorken & Glashow predicted the existence of the charm quark

$$J_{\mu, h}^+ = \bar{u} \gamma_\mu (1 - \gamma_5) d' + \bar{c} \gamma_\mu (1 - \gamma_5) s'$$

$$s' = -\sin\theta_c d + \cos\theta_c s$$

Comparing to  $\beta$ -decay, Fermi found

$$G_F \sim 10^{-5} m_p^{-2}$$

Fermi's Theory (updated to V-A) accurately predicts many observed Weak interactions

$$n \rightarrow p e^- \bar{\nu}_e, \quad \bar{\nu}_e n \rightarrow p e^-, \quad e^- p \rightarrow n \bar{\nu}_e$$

$$\mu^- \rightarrow e^- \gamma \bar{\nu}_e$$

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

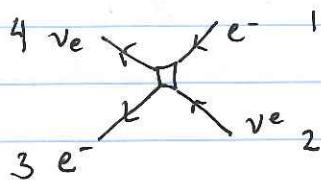
$$e^- \bar{\nu}_e$$

$$\Lambda \rightarrow p \pi^-$$

:

But there are well known problems w/ Fermi's theory.

eg.  $\nu_e e^- \rightarrow \nu_e e^-$  scattering



$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \bar{e} \gamma^\mu (1-\gamma_5) \nu_e$$

$$iA = -i \frac{G_F}{\sqrt{2}} \bar{u}_4 \gamma_\mu (1-\gamma_5) u_1 \bar{u}_3 \gamma^\mu (1-\gamma_5) u_2$$

$\underbrace{\quad \quad \quad}_{\text{spins unrelated}}$

unpolarized, spin-averaged

$$|A|^2 = \frac{G_F^2}{2} \cdot \frac{1}{2} \text{Tr} \left[ \bar{u}_1 \gamma_\mu (1-\gamma_5) u_4 \bar{u}_4 \gamma_\nu (1-\gamma_5) u_1 \right]$$

$$\times \text{Tr} \left[ \bar{u}_2 \gamma^\mu (1-\gamma_5) u_3 \bar{u}_3 \gamma^\nu (1-\gamma_5) u_2 \right]$$

$$\approx \frac{G_F^2}{4} \text{Tr} \left[ \not{p}_1 \gamma_\mu (1-\gamma_5) \not{p}_4 \gamma_\nu (1-\gamma_5) \right]$$

$$+ \text{Tr} \left[ \not{p}_2 \gamma^\mu (1-\gamma_5) \not{p}_3 \gamma^\nu (1-\gamma_5) \right]$$

$$\approx \frac{G_F^2}{4} \text{Tr} \left[ \not{p}_1 \gamma_\mu \not{p}_4 \gamma_\nu + \not{p}_1 \gamma_\mu \not{p}_5 \not{p}_4 \gamma_\nu \gamma_5 - \not{p}_1 \gamma_\mu \not{p}_5 \not{p}_4 \gamma_\nu - \not{p}_1 \gamma_\mu \not{p}_4 \gamma_\nu \gamma_5 \right]$$

$$\times \text{Tr} \left[ \right]$$

$$\approx \frac{G_F^2}{4} \text{Tr} \left[ 2 \not{p}_1 \gamma_\mu \not{p}_4 \gamma_\nu - 2 \not{p}_1 \gamma_\mu \not{p}_4 \gamma_\nu \gamma_5 \right] \text{Tr} \left[ 2 \not{p}_2 \gamma^\mu \not{p}_3 \gamma^\nu - 2 \not{p}_2 \gamma^\mu \not{p}_3 \gamma^\nu \gamma_5 \right]$$

$$= 64 G_F^2 \cdot p_1 \cdot p_2 \cdot p_3 \cdot p_4 \quad (p_1 \cdot p_2 = \frac{1}{2} s - p_1^2 - p_2^2)$$

$$|A|^2 = 16 G_F^2 s^2$$

$$\frac{d\sigma}{d\cos\theta} = \frac{|A|^2}{32\pi s}$$

$$= \frac{G_F^2 s}{2\pi}$$

$$\sigma = \int_{-1}^1 \frac{d\cos\theta}{d\cos\theta} d\cos\theta = \frac{G_F^2 s}{\pi}$$

Note This has the  
correct dimensions for  
a cross section

The problem is this cross section violates unitarity.

Notice the cross section was pure s-wave.

Unitarity:  $S^\dagger S = 1$  (sum of probabilities of everything that can happen must = 1)

$$S = 1 + i T$$

$$\begin{aligned} S^\dagger S &= (1 - iT^\dagger)(1 + iT) \\ &= 1 - i(T^\dagger - T) + T^\dagger T \end{aligned}$$

2a

$$i(T^\dagger - T) = T^\dagger T$$

$$2i\text{Im } T = |T|^2$$

$$= 4k\sqrt{s} \sigma(k)$$

$$\Rightarrow \sigma \leq \frac{16\pi}{s}$$

$$\frac{G_F^2 S}{\pi} \leq \frac{16\pi}{S}$$

$$S^2 \leq \frac{16 n^2}{C_F^2}, \quad S = 4 E_{cm}^2$$

$$E_{CM}^2 \leq \frac{4\pi}{G_F}$$

1. What is the difference between a primary and secondary market?

卷之三

✓ 100% 100% 100%

二十一

卷之三

$$G_F \approx 1.13 \times 10^{-7} GeV^{-2}$$

or, for  $E_{cm} > 2\sqrt{\frac{\mu}{G_F}} \sim 1000 \text{ GeV}$

unitarity is violated.

This is not a total disaster. We have only computed the LO term.

In perturbation theory, unitarity is often not restored until we compute higher order terms.

$$\text{Diagram} = \text{Diagram} + \text{Diagram} + \text{Diagram} + \dots$$

$\text{Im } A \neq 0$ , Intermediate particles going on-shell produces a cut, and hence imaginary part of amplitude.

Diagrammatically, we see

$$2 \operatorname{Im} X = |X|^2$$

Now, in Fermi's theory

$$\text{Diagram} \sim \int d^4 k \left( \frac{k}{k^2} \right)^2 \sim \Lambda^2$$

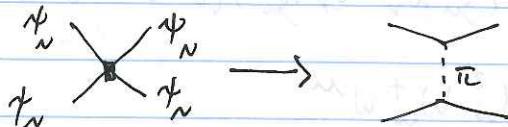
This contribution requires new operators due<sup>2</sup> to the non-renormalizable nature of the theory

In the 60's & 70's when these issues were

being resolved, we believed nature was described only by renormalizable QFT.

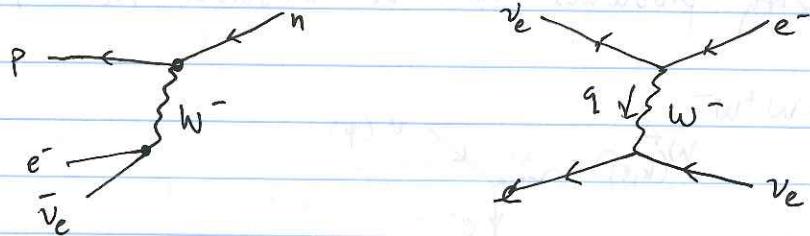
- We already had the idea of replacing 4-fermion interactions with intermediate meson exchange

Yukawa 1935  
(Schwinger 1957)



For the Weak interactions, the idea was to add a massive spin-1 vector boson (Glashow)

$$\mathcal{L} = g W_\mu^+ J^\mu + g W_\mu^- J^\mu$$



Why massive?

$$m \sim \frac{1}{q^2 - M_W^2} = \frac{1}{M_W^2} \frac{1}{1 + Q^2/M_W^2}, \quad Q^2 = -q^2$$

for t-channel,  $Q^2 < 0$  (space-like)

For  $Q^2 \ll M_W^2$

Feynman diagram for the t-channel exchange of a  $W^-$  boson between a proton and an electron-antineutrino, showing its equivalence to a contact interaction with coupling  $G_F$ . The diagram is equivalent to a contact interaction with coupling  $G_F \sim \frac{g^2}{M_W^2} \sim \frac{1}{10^5 m_p^2}$ .